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Physical Properties of Seawater: A New Salinity Scale and Equation of State for Seawater

N. P. FOFONOFF

Woods Hole Oceanographic Institution, Massachusetts

Developments leading to the new practical salinity scale and equation of state for seawater, introduced by the Joint Panel on Oceanographic Tables and Standards (JPOTS), are reviewed. The laboratory measurements used to construct the new empirical formulas for converting conductivity ratios to salinity and for estimating seawater density from salinity, temperature, and pressure were made on samples of standard seawater to minimize the effects of compositional variations. Measured densities were fitted with a standard deviation of 0.0036 kg/m^3 at atmospheric pressure and 0.009 kg/m^3 over the oceanic pressure range. Natural seawater densities can differ from computed density by as much as 0.05 kg/m^3 because of variations in the composition of dissolved salts that are not reflected equivalently in measured conductivity ratios. Differences from the previous Knudsen-Ekman equation of state are also of comparable magnitude. Values of specific volume anomaly for geopotential anomaly computations for estimation of geostrophic currents and transport are not significantly modified. Algorithms adopted by the JPOTS for computing adiabatic lapse rate and potential temperature are compared with formulas derived from the new equation of state. Potential temperatures agree within 2×10^{-3}°C over the oceanic range of temperature and pressure at salinity of 35.

HISTORICAL BACKGROUND

The completion in 1981 of a series of recommendations and reports by the Joint Panel on Oceanographic Tables and Standards (JPOTS) on the definition of a practical salinity scale (PSS 78) and an equation of state for seawater (EOS 80) marked a milestone in the establishment of precise and reproducible standards for salinity, density, and related variables for ocean waters.

Because JPOTS introduced a new salinity scale based on constant composition of salts and expressed in terms of an electrical conductivity ratio, consistent algorithms had to be developed for other derived or dependent functions. Formulas for a few of the more commonly used physical properties have been tabulated by *Fofonoff and Millard* [1984]. These are discussed in the present article together with a review of some of the considerations leading to the new definitions and algorithms. The notation and definitions are similar to those used by *Fofonoff* [1962]. The units and symbols follow recommendations of the IAPSO (International Association for Physical Sciences of the Oceans) working group on symbols, units, and nomenclature [*IAPSO*, 1979].

The evolution of the practical salinity scale and equation of state was initiated in part by *Eckart*'s [1958] concern about the accuracy of knowledge of fundamental properties such as density and thermal expansion of seawater. His examination of the available data led him to conclude that thermal expansion coefficients of seawater and even pure water were not known with acceptable accuracy. With the cooperation and support of the U.S. National Academy of Sciences Committee on Oceanography and the Office of Naval Research, he convened a conference to examine the state of knowledge of the chemical and physical properties of seawater with the aim of stimulating and focusing further research in this area. Among the recommendations included in the conference report [*NAS-NRC*, 1958] were steps to label and certify Copenhagen Standard Seawater for electrical conductivity and density as well as chlorinity. These recommendations were made in recognition of the increasing use of conductivity bridges (salinometers) for salinity determination and increasing concern about the adequacy of a chlorinity standard to serve as a conductivity standard.

At the same conference, *Reid* [1958] presented a comprehensive analysis showing the dependence of geostrophic and dynamic computations on errors in the equation of state. He computed the sensitivity to thermal expansion and the saline contraction coefficients to demonstrate that dynamic computations were not limited by inaccuracies of the coefficients. For other calculations, such as stability or Brunt-Väisälä frequency, accurate coefficients are a necessity.

A study of the relationships among chlorinity, conductivity, and density was undertaken at the British National Institute of Oceanography in 1960 on several hundred samples of seawater collected from widely distributed locations over the world oceans. *Cox et al.* [1962] reported a markedly higher correlation between density and conductivity than between density and chlorinity. These findings tended to confirm the misgivings about the methods for standardizing salinity measurements and led to the formation of a Joint Panel on the Equation of State of Seawater, sponsored by ICES (International Council for the Exploration of the Sea), IAPSO, SCOR (Scientific Committee on Oceanic Research) and UNESCO (renamed the Joint Panel on Oceanographic Tables and Standards (JPOTS) in 1964). The panel's task was to review the relationship of the equation of state to chlorinity, salinity, electrical conductivity, and refractive index to see if redefinition of any of these properties was necessary. The panel members concluded, after reviewing the data presented by *Cox* [1962; reprinted in *UNESCO*, 1976*a*, Append. 1], that salinity should not be defined in terms of chlorinity because of the excessive variability of its relationship to density. Although some of the variability later proved to be spurious and a consequence of the laboratory technique used to determine chlorinity [*Cox et al.*, 1967], the scatter consistently exceeded the precision of measurement, supporting the possibility that density could be estimated more precisely from conductivity than from chlorinity.

Because of the better correlation between conductivity and

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density, the initial panel recommendation was to establish an equation defining salinity in terms of density. Different techniques of measurement could then be implemented by construction of formulas relating density to a measured variable such as electrical conductivity, refractive index, or density itself. Because density can be measured, whereas salinity cannot, the link to salinity would be through the defining density equation. Salinity must be proportional to chlorinity to retain its conservative property and to have a proportionality factor of 1.80655 to match historical data. Subsequent determinations of salinity of standard seawater (SSW) would be made from measurements of density or conductivity but not chlorinity, which would be reported as an independent variable.

The recommendations were made with the assumption that absolute density and conductivity of SSW could be measured with the required precision. However, technical problems encountered in constructing equipment to achieve the precision necessary for acceptable standards delayed implementation of the recommendations. Relative measurements were considered and dismissed because the references could not be defined with sufficient precision. Distilled water, for example, differed in density, depending on its origin and method of preparation. The uncertainties in density were believed to be as large as 30 parts per million [*UNESCO*, 1976*a*, Append. 2]. Similarly, conductivity references, such as potassium chloride solutions, required temperature regulation that was not readily achievable [*Cox*, 1962]. As a result, only part of the recommendations could be implemented. Salinity was defined in terms of the electrical conductivity ratio R_{15} at 15°C to seawater having a salinity of 35 [*UNESCO*, 1966, 1976*b*]. The reference seawater salinity was not defined separately in terms of conductivity ratio or density.

Tables to convert conductivity ratio to salinity, based on a regression formula developed by *Cox* [1962], were published by *NIO-UNESCO* [1966] and widely distributed to oceanographers with the understanding that SSW would be certified for conductivity as soon as the necessary equipment was completed. The new scale was officially adopted after Cox's death in 1967 and was formally announced in 1969 [*Wooster et al.*, 1969].

Objections to the 1969 scale were raised. *Park* [1964] had already noted that conductivity of different batches of SSW measured relative to a single reference batch varied in excess of the precision of conductivity measurements and had called for certification of conductivity as well as chlorinity for SSW.

Other objections surfaced. The determinations of R_{15} [*Cox et al.*, 1967] were made on natural seawater samples of variable composition. Some midrange salinities were obtained by mixing Baltic and Red Sea samples. As the salinity-chlorinity conversions assumed fixed composition, the salinities were not accurately conservative. As salt ratios of natural seawaters are time dependent, especially at low salinities, the scale is not accurately reproducible. Samples from the same geographical locations could not be relied upon to yield the same relationship between conductivity and chlorinity. The increasing use of in situ instruments measuring conductivity forced the use of conversion formulas that were obtained by using standard seawater diluted with distilled water [*Brown and Allentoft*, 1966]. These formulas were not consistent with the JPOTS versions, which did not cover the temperature range below 10°C. Some of the problems associated with the 1969 scale were addressed in the sixth JPOTS report [*UNESCO*, 1974] with recommendations to measure the dependence of conductivity on temperature and pressure. Also, to have an independent check on the absolute conductivity apparatus, Poisson was encouraged to make some measurements of electrical conductivity of standard seawater samples relative to potassium chloride solutions to compare with the absolute measurements planned at National Institute of Oceanography (U.K.) by Culkin. Poisson was able to report at a subsequent JPOTS meeting [*UNESCO*, 1976*a*] that deviations of conductivity equivalent to 0.006 salinity were found for different batches of SSW. The panel viewed these results with some consternation and called for confirmatory studies. Several laboratories were asked subsequently to measure conductivity of 26 batches of SSW relative to P64. These studies [*Millero et al.*, 1977] confirmed Poisson's findings. Some of the batches were high in conductivity by amounts equivalent to 0.004 to 0.007 salinity [*UNESCO*, 1978]. Poisson reported at the same meeting of JPOTS that potassium chloride solutions could be made with high and reproducible accuracy. Furthermore, the temperature control required for thermostatted baths was well within the capability of commercially available equipment. Thus the reservations about accepting potassium chloride as a conductivity standard were removed. The panel accepted a recommendation to use potassium chloride solution as a standard and outlined a procedure to establish the salinity scale. The determinations were to be made on a single batch of SSW to ensure constancy of composition. The scale was to be related to the 1969 and previous scales at 35 by the chlorinity factor 1.80655 for that particular batch. For subsequent batches the salinity was to be determined from the conductivity ratio to the reference KCl standard and not from chlorinity. Hence the chlorinity factor is not a constant but would vary slightly from batch to batch.

The steps that followed to define the practical salinity scale are thoroughly documented in the eighth, ninth, and tenth reports of JPOTS [*UNESCO*, 1978, 1979, 1981*c*] and collected background papers [*UNESCO*, 1981*b*].

In addition to the definition of the salinity scale, the panel encouraged the development of algorithms for converting in situ measurements of conductivity ratio to salinity. These algorithms were developed to be consistent with the new practical salinity.

THE PRACTICAL SALINITY SCALE (PSS 78)

Prior to the introduction of the 1969 salinity scale, salinity S was expressed in terms of chlorinity Cl, defined as the chloride-equivalent mass ratio of halides to the mass of seawater, by a linear least squares regression formula fitted to mass ratios of salt residues of nine seawater samples evaporated to dryness [*Knudsen et al.*, 1902]. The relationship $S = 0.030 + 1.8050$ Cl served oceanographers for over six decades. The formula for S, commonly referred to as the Knudsen salinity, incorporated changing composition of seawater with dilution and consequently was not conservative with respect to addition or removal of water. As long as salinities were computed from chlorinities, the Knudsen scale was acceptable. The increasing use, during the 1950's, of high-precision electrical conductivity bridges for salinity determinations created the need for new standards for salinity measurements.

The 1969 scale, proportional to chlorinity to satisfy mass conservation, was chosen to coincide with the Knudsen scale at salinity of 35, yielding the factor $1.80655 = 1.8050[35/(35 - 0.030)]$. Differences between the two scales are less than 0.003 over the salinity range 32–38 and increase to 0.03 at low salinity. As noted earlier, the 1969 scale was

TABLE 1. Concentration of Potassium Chloride Solutions Having the Same Conductivity as Seawater (Batch P79) at Salinity $S = 35$ (PSS 78)

Reference	Concentration g/kg	Standard Deviation of Residuals, g/kg	Number of Measurements	Standard Deviation of Mean, g/kg
Dauphinee et al. [1980*a*]	32.4356	3.6×10^{-4}	21	0.8×10^{-4}
Poisson [1980*a*]	32.4358	4.7×10^{-4}	14	1.3×10^{-4}
Culkin and Smith [1980]	32.4352	(3.4×10^{-4})	17	(0.8×10^{-4})
$S = 35$ (PSS 78)	32.4356	0	(by definition)	

Values in parentheses are calculated from published table.

deficient in not establishing a salinity reference point independent of chlorinity and inconsistent in using seawater samples of variable composition to relate salinity to electrical conductivity.

The fundamental step in constructing the practical salinity scale PSS 78 consisted of defining a single reference point ($S = 35$) on the scale as having the same electrical conductivity as a reference potassium chloride (KCl) solution at 15°C and atmospheric pressure. The transition from the previous scale was made by selecting a single batch (P79) of SSW and equating the new scale to the old through the chlorinity relationship $S = 1.80655$ Cl for that particular batch. As the salinity on the practical scale is defined to be conservative with respect to addition and removal of water, the entire salinity range is accessible through precise weight dilution or evaporation without additional definitions. However, the practical scale is defined in terms of conductivity ratio and not by its conservative properties. Thus the second part of the definition was constructed by measuring the conductivity ratio to the KCl standard or an equivalent secondary standard over the entire range of salinities (1 to 42) of samples prepared by evaporation or dilution of batch P79 SSW and computing an empirical formula $S = S(R_{15})$, where R_{15} is the conductivity ratio at 15°C and atmospheric pressure to the KCl standard. Thus salinities on the PSS 78 scale are defined by conductivity ratios alone. Salinities determined by any other method would not necessarily coincide with the PSS 78 scale and would have to be identified separately. *Lewis and Perkin* [1981] discuss differences between PSS 78 and previous scales and provide algorithms and tables to convert existing data to the new scale.

The KCl reference solution for $S = 35$ was established by three independent sets of measurements by *Dauphinee et al.* [1980*a*], *Poisson* [1980*a*], and *Culkin and Smith* [1980]. The values obtained and the precision of the determinations are summarized in Table 1. Both standard deviations of residuals and of the means are computed. The means given by *Dauphinee et al.* [1980*a*] and *Poisson* [1980*a*] are not significantly different from each other but are significantly different (3 to 4 standard deviations) from the mean value obtained by Culkin and Smith. Thus, although all three determinations are sufficiently accurate to define the reference an order of magnitude more precisely than required for routine CTD instruments, statistically significant differences did appear for the absolute measurements of Culkin and Smith.

The data sets used by *Perkin and Lewis* [1980] to express the dependence of conductivity ratio on temperature and pressure are listed in Table 2. All of the measurements were made by using PSS 78 and batches P73, P75, and P79 of SSW. Differences in composition between the batches would have negligible effects on the temperature and pressure coefficients of conductivity ratios, as neither are strong functions of salinity. *Mantyla* [1980] measured the conductivity ratios of 39 different batches of SSW relative to P79. Batches P73 and P75 yielded differences of −0.0011 and +0.0005, respectively, between conductivity and chlorinity salinity. As the standard deviation of replicates was 0.0004, the batches were not distinguishable from one another in composition.

The algorithm for converting conductivity ratio to salinity is constructed in terms of the conductivity ratio R, defined as

$$R = C(S, t, p)/C(35, 15, 0) \tag{1}$$

TABLE 2. Precision of Empirical Formulas Relating Salinity and Conductivity Ratios R_t, r_t, and R_p

Reference	Standard Deviation of Residuals, (Salinity Equivalent)	Number of Observations	SSW Batch
	$S = S(R_{15})$		
Dauphinee et al. [1980*a*]	3.6×10^{-4}	116	P79
Poisson [1980*a*]	4×10^{-4}	38	P75
Perkin and Lewis [1980]	4.8×10^{-4}	combined fit	
	$S = S(t, R_t)$		
Dauphinee et al. [1980*b*]	—	189	P79
Poisson [1980*b*]	10×10^{-4}	115	P75
Bradshaw and Schleicher [1980]	8×10^{-4}	101	P75
Perkin and Lewis [1980]	6.6×10^{-4}	combined fit	
	$r_t = r_t(t)$		
Dauphinee et al. [1980*b*]	4.4×10^{-4}	34	P79
Bradshaw and Schleicher [1980]	3×10^{-4}	36	P73, P75
Perkin and Lewis [1980]	3.2×10^{-4}	combined fit	
	$R_p = R_p(R, t, p)$		
Bradshaw and Schleicher [1980]	13×10^{-4}	224	P75

where $C(S, t, p)$ is electrical conductivity as a function of salinity S, temperature t, and pressure p. The ratio is factored into the functions

$$R = r_t(t)R_t(S, t)R_p(R, t, p) \tag{2}$$

where

$$r_t(t) = C(35, t, 0)/C(35, 15, 0)$$

$$R_t(S, t) = C(S, t, 0)/C(35, t, 0)$$

$$R_p(R, t, p) = C(S, t, p)/C(S, t, 0)$$

JPOTS has not accepted or recommended a value for conductivity $C(35, 15, 0)$ to avoid any confusion with conductivity ratio in the definition of practical salinity. For those applications where absolute conductivity is required, a value of 4.29140 S/m (42.9140 mmho/cm) can be used to convert conductivity ratio to electrical conductivity. This value was obtained by *Culkin and Smith* [1980] using absolute conductivity equipment at the British Institute of Ocean Sciences.

Salinity is given by the function

$$S = \sum_{n=0}^{5} \left(a_n + \frac{\Delta t}{1 + k\Delta t} b_n\right) R_t^{n/2} \tag{3}$$

with coefficients

$$\begin{array}{lll} a_0 = +0.0080 & b_0 = +0.0005 & k = +0.0162 \\ a_1 = -0.1692 & b_1 = -0.0056 & \Delta t = t - 15 \\ a_2 = +25.3851 & b_2 = -0.0066 & \\ a_3 = +14.0941 & b_3 = -0.0375 & \\ a_4 = -7.0261 & b_4 = +0.0636 & \\ a_5 = +2.7081 & b_5 = -0.0144 & \end{array}$$

At $t = 15°\mathrm{C}$, (3) reduces to the formula defining the practical salinity scale [*Perkin and Lewis*, 1980].

The factors r_t and R_p are given by

$$r_t = \sum_{n=0}^{4} C_n t^n \tag{4}$$

where

$$\begin{array}{ll} C_0 = +0.6766097 & C_3 = -6.9698\mathrm{E}{-7} \\ C_1 = +2.00564\mathrm{E}{-2} & C_4 = +1.0031\mathrm{E}{-9} \\ C_2 = +1.104259\mathrm{E}{-4} & \end{array}$$

and

$$R_p = 1 + \frac{e_1 p + e_2 p^2 + e_3 p^3}{1 + d_1 t + d_2 t^2 + (d_3 + d_4 t)R} = 1 + \frac{C}{B + AR} \tag{5}$$

with

$$\begin{array}{ll} e_1 = +2.070\mathrm{E}{-5} & d_1 = +3.426\mathrm{E}{-2} \\ e_2 = -6.370\mathrm{E}{-10} & d_2 = +4.464\mathrm{E}{-4} \\ e_3 = +3.989\mathrm{E}{-15} & d_3 = +4.215\mathrm{E}{-1} \\ & d_4 = -3.107\mathrm{E}{-3} \end{array}$$

for temperature in degrees Celsius and pressure in decibars.

Given a measurement of R, t, and p, salinity is computed by solving (2) for R_t; $R_t = R/(r_t R_p)$ and evaluating S from (3). If conductivity ratio R is required, given S, t, and p, the ratio R_t can be found by numerical inversion of (3), and R can be found by solving the quadratic equation

$$R = r_t R_t R_p = r_t R_t \left(1 + \frac{C}{AR + B}\right)$$

or

$$R = \{[(Ar_tR_t - B)^2 + 4r_tR_tA(B + C)]^{1/2} + [Ar_tR_t - B]\}/2A \tag{6}$$

More detailed descriptions, including FORTRAN programs, check values, limits of validity, and tables are given by *Fofonoff and Millard* [1984].

IDEAL OR ABSOLUTE SALINITY

Absolute or ideal salinity, defined simply as the mass fraction of salts in seawater, is important conceptually for formulating rigorous descriptions and precise assumptions about the dynamical and thermodynamical behavior of a seawater system. In its 1978 recommendations, JPOTS proposed use of the symbol S_A to denote absolute salinity to distinguish it from practical salinity S. The practical salinity must be proportional to S_A (or s used here) to represent conservation or salt mass in seawater.

The aggregate variable, salinity s, is useful because major ions are in nearly constant proportions to one another throughout the world oceans. This "law of constant proportions," first enunciated by *Marcet* [1819] and elaborated and documented more quantitatively by *Forchhammer* [1865] and *Dittmar* [1884], appears to hold, with some exceptions, to within experimental error [*Culkin*, 1965]. However, the error of determining some of the major constituents separately is relatively large, so that a rigorous test to the same precision as conductivity measurements is difficult to carry out. Exceptions to the constancy of composition are found in coastal and enclosed regions, such as the Baltic Sea, diluted by land drainage and surface waters in which biological processes can alter the composition significantly. *Culkin and Cox* [1966] noted that the concentration of calcium is the major variable component affecting density and conductivity between surface and deep waters of the open ocean. As real variations in composition are present, the effects of salt content in seawater cannot be completely described by a single variable (salinity) within the framework of the hydrodynamic equations. The ideal salinity has limitations that require discussion. The first is that no convenient, inexpensive, and accurate techniques exist for determining total salt content in seawater. Estimates of salinity, obtained indirectly from measurements of chlorinity, electrical conductivity and, less commonly, specific gravity or refractive index, yield estimates of salinity on practical scales, such as PSS 78 discussed earlier, whose precise relationships to absolute salinity are unknown. The second is that an advective-diffusive conservation equation cannot be written for salinity alone because diffusion does not change the salt distribution toward constant composition or uniform salinity. The law of constant proportions is a consequence of dynamical processes of advection and turbulent mixing rather than a trend to thermodynamical equilibrium. This conclusion is developed further by examining the conservation equations for a multicomponent thermodynamical system.

Suppose that a mass m_i of the ith solute constituent is present in a mass m of seawater consisting of m_w water and $m_s =$

$\sum m_i$ of solutes. The mass fraction of solutes or ideal salinity s is defined to be

$$s = m_s/(m_w + m_s) = m_s/m \tag{7}$$

and the partial salinity (concentration) $s_i = m_i/m$ of the ith solute so that

$$s = \sum s_i \tag{8}$$

If λ_i is the ratio of the ith constituent to the total solutes so that $s_i = \lambda_i s$, the law of constant proportions implies that λ_i is independent of s.

A mass conservation equation must hold for each constituent m_i. If ρ_i is the partial density of the ith constituent and $\mathbf{V}_i$ its velocity, the individual conservation equations take the form

$$\begin{aligned} \frac{\partial \rho_i}{\partial t} + \nabla \rho_i \mathbf{V}_i &= 0 \\ \frac{\partial \rho_w}{\partial t} + \nabla \rho_w \mathbf{V}_w &= 0 \end{aligned} \tag{9}$$

where the subscript w denotes the water or solvent component. The sum of the individual conservation equations leads to the usual continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \rho\ \mathbf{V} = 0 \tag{10}$$

provided

$$\begin{aligned} \rho &= \sum \rho_i + \rho_w = \rho\left(\textstyle\sum s_i + s_w\right) \\ \mathbf{V} &= \left(\textstyle\sum \rho_i \mathbf{V}_i + \rho_w \mathbf{V}_w\right)/\rho \\ &= \textstyle\sum s_i \mathbf{V}_i + s_w \mathbf{V}_w \end{aligned} \tag{11}$$

are defined as the density ρ and velocity $\mathbf{V}$ of seawater. The concentrations s_i of solutes and s_w of water are related by

$$\sum s_i + s_w = 1 \tag{12}$$

The individual equations can be rewritten relative to the mean velocity $\mathbf{V}$ as

$$\frac{\partial \rho_i}{\partial t} + \nabla \rho_i \mathbf{V} = -\nabla \rho(\mathbf{V}_i - \mathbf{V}) = -\nabla \mathbf{F}_i \tag{13}$$

where

$$\mathbf{F}_i = \rho_i(\mathbf{V}_i - \mathbf{V}) = \rho s_i(\mathbf{V}_i - \mathbf{V}) \tag{14}$$

represents the flux or diffusion of the ith solute relative to the total mass flux. The driving force for diffusion of the ith component is assumed to be the gradient of its partial chemical potential $\nabla\mu_i$. If the flux is denoted by $\mathbf{J}_i = \rho s_i \mathbf{V}_i$, the product $\mathbf{J}_i \cdot \nabla\mu_i$ represents the rate of energy conversion for the ith component associated with the flux. The sum $(\sum \mathbf{J}_i \cdot \nabla\mu_i + \mathbf{J}_w \cdot \nabla\mu_w)$ represents a rate of energy conversion D. As only the relative or diffusion fluxes contribute, the conversion can be interpreted as a dissipation rate of energy D or production of entropy D/T, where T is absolute temperature. Using the Gibbs-Duhem equation [*Katchalsky and Curran*, 1967] and neglecting thermal effects, the dissipation rate for fluxes $\mathbf{F}_i$, relative to the mass flux $\rho\mathbf{V}$, can be put in the form

$$\begin{aligned} D &= -\sum \mathbf{J}_i \cdot \nabla\mu_i - \mathbf{J}_w \cdot \nabla\mu_w \\ &= -\sum \mathbf{F}_i \cdot \nabla(\mu_i - \mu_w) \end{aligned} \tag{15}$$

For diffusive fluxes the entropy production on dissipation is a positive definite function. The fluxes will be linear functions of the form

$$\mathbf{F}_i = \sum_k \nu_{ik} \nabla(\mu_k - \mu_w) \tag{16}$$

The effects of diffusion are to diminish the gradients of $\mu_i - \mu_w$ at rates that can vary from solute to solute. *Caldwell and Eide* [1981] found variations of a factor of 2 or more in the diffusivities of major salts in seawater. Although diffusive fluxes must be further constrained and modified by requirement of balance of electrical charges and by variations of pressure and temperature, it is clear that diffusion does not act to preserve constancy of salt ratios but rather to equilibrate partial chemical potentials.

Fofonoff [1962] estimated that the salinity gradient required to equalize the pressure dependence of chemical potential difference $\mu = \sum \lambda_i\mu_i - \mu_w = \mu_s - \mu_w$ is about 3 to 4 per 1000 decibars. As vertical gradients of this magnitude are not observed in the ocean except over short vertical scales, the diffusive fluxes do not appear to play a significant role compared with advective and turbulent transports. The representation of diffusive fluxes in seawater proportional to gradients of salinity is, therefore, misleading and, except for the smallest scales, incorrect because the thermodynamical equilibrium state is not incorporated into the equations and cannot be in terms of salinity alone. The different diffusion rates can become important if the diffusion time scales are comparable or exceed the advective scales.

EQUATION OF STATE FOR SEAWATER

Natural variability in the composition of seawater results in fluctuations in the relationship between density and conductivity that exceed the resolution of measurement. The density of a sample of natural seawater cannot be specified by salinity alone to better than about 50×10^{-3} kg/m^3 unless corrections for composition are made. *Brewer and Bradshaw* [1975] estimated that the effects of increased alkalinity, total carbon dioxide, and silica content on the conductivity-density relationship lead to an underestimate of density from conductivity salinity alone by about 12×10^{-3} kg/m^3 for the deep North Pacific waters. Such a correction to density is equivalent to the difference of potential density over 500 m vertically. *Millero et al.* [1978] found differences of 17.6×10^{-3} kg/m^3 between measured densities and densities calculated from conductivity salinity at depths below 1000 m for the North Pacific. *Poisson et al.* [1981] reported differences of measured and calculated densities for surface waters ranging from -6×10^{-3} kg/m^3 in the western Indian Ocean to -35×10^{-3} kg/m^3 in the Red Sea.

A precise equation of state, reproducible to better than 10×10^{-3} kg/m^3, requires not only a precise definition of salinity but also of water itself. Both chemical and isotopic variations in natural waters and seawaters affect density at this level of precision. JPOTS had first to define a salinity scale and to choose a reference pure water of specified isotopic composition before considering candidate formulas for the density and specific volume. Procedures for defining the practical salinity scale and the equation of state were set out in the eighth report [*UNESCO*, 1978] of the panel. Five data sets were identified as the basis for developing the equation of state, using the general form adopted by *Chen and Millero* [1976], similar to the earlier equation used by *Ekman* [1908]. Ekman incorporated the effects of pressure in a mean compression coefficient, whereas Chen and Millero fitted the mean

TABLE 3. Coefficients for the International Equation of State for Seawater EOS 80

	A	B	C	D
t^0	+999.842594*	+8.24493E-1	−5.72466E-3	+4.8314E-4
t^1	+6.793952E-2	−4.0899 E-3	+1.0227 E-4	
t^2	−9.095290E-3	+7.6438 E-5	−1.6546 E-6	
t^3	+1.001685E-4	−8.2467 E-7		
t^4	−1.120083E-6	+5.3875 E-9		
t^5	+6.536332E-9			
	E	**F**	**G**	
t^0	19652.21*	+54.6746	+7.944 E-2	
t^1	+148.4206	−0.603459	+1.6483E-2	
t^2	−2.327105	+1.09987E-2	−5.3009E-4	
t^3	+1.360477E-2	−6.1670 E-5		
t^4	−5.155288E-5			
	H	**I**	**J**	
t^0	+3.239908*	+2.2838E-3	+1.91075E-4	
t^1	+1.43713E-3	−1.0981E-5		
t^2	+1.16092E-4	−1.6078E-6		
t^3	−5.77905E-7			
	M	**N**		
t^0	+8.50935E-5*	−9.9348E-7		
t^1	−6.12293E-6	+2.0816E-8		
t^2	+5 2787 E-8	+9.1697E-10		

$$
\begin{aligned}
V(S, t, p) &= V(S, t, 0)[1 - p/K(S, t, p)] \\
\rho(S, t, 0) = 1/V(S, t, 0) &= A + BS + CS^{3/2} + DS^2 \\
K(S, t, p) &= E + FS + GS^{3/2} \\
&\quad + (H + IS + JS^{3/2})P \\
&\quad + (M + NS)p^2
\end{aligned}
$$

where the coefficients are polynomials in temperature t. Units: salinity, PSS 78; temperature, °C; pressure, bars; density, kg/m^3; specific volume, m^3/g.

*Coefficients modified for calculation of steric anomaly (Table 4).

or secant bulk modulus, the reciprocal of mean compression, to the data.

The data sets consist of measurements made with a magnetic float densimeter of the density difference between seawater and pure water at atmospheric pressure [*Millero et al.*, 1976] and at elevated pressures [*Chen and Millero*, 1976]; measurements of compression and thermal expansion by *Bradshaw and Schleicher* [1970, 1976]; and compressibility coefficients estimated from sound speed measurements of *Chen et al.* [1977] and *Chen and Millero* [1978]. A sixth data set was added [*UNESCO*, 1979; *Poisson et al.*, 1980] to improve the accuracy of the equation at atmospheric pressure at salinity of 35 and temperatures above 25°C. The revised formula was completed by *Millero and Poisson* [1981] and recommended by JPOTS in the tenth report [*UNESCO*, 1981*a*].

Densities of seawater are not determined relative to absolute standards of mass and length but rather as differences from a defined pure water standard of known isotopic composition. *Craig* [1961] described a standard mean ocean water (SMOW) with isotopic composition referred to a National Bureau of Standards reference sample. Menache [*Girard and Menache*, 1972; *UNESCO*, 1974] proposed that SMOW be used as the density reference with a provisional value of the maximum density of 999.975 kg/m^3, and the density variation with temperature given by a fifth-order polynomial fitted to *Bigg*'s [1967] table of pure water densities. Bigg's table was computed as a weighted mean of measurements made by *Chappuis* [1907] and *Thiesen et al.* [1900] on pure water. Bigg adjusted the earlier values to take into account the redefinition of the liter in 1964. The polynomial fitted to Bigg's values was scaled by a constant factor to adjust the maximum density from 999.972 to 999.975 kg/m^3 for SMOW. Menache [*UNESCO*, 1976*a*, Annex 5] estimated that the uncertainty in the maximum density of SMOW did not exceed 3×10^{-3} kg/m^3. The uncertainty is of little consequence in most oceanographic work, provided the same reference is used consistently.

Density differences between seawater and SMOW were computed from the data of *Millero et al.* [1976] and *Poisson et al.* [1980]. Polynomials in temperature and salinity (half powers) were fitted to the differences to yield a 1-atmosphere equation with a standard error (rms residuals) of 3.6×10^{-3} kg/m^3 [*Millero and Poisson*, 1981].

The pressure dependence was developed as a mean bulk modulus for pure water K_w based on *Kell*'s [1975] and isothermal compressibilities near 1 atmosphere from sound speed measurements of *Del Grosso* [1970] and *Del Grosso and Mader* [1972]. The bulk modulus for seawater was determined from sound speed measurements of *Millero and Kubinski* [1975]. At higher pressures the differences from pure water, $V(S, t, p) - V(O, t, p)$, were calculated from measurements by *Chen and Millero* [1977] and *Bradshaw and Schleicher* [1976]. The differences were then fitted by a polynomial in salinity (half powers), temperature, and pressure [*Millero et al.*, 1980]. The overall standard deviation for $V(S, t, p)$ is 9.0×10^{-9} m^3/kg. The final equation of state was adopted by JPOTS in 1981 [*UNESCO*, 1981*a*]. Coefficients for the complete equation are given in Table 3.

DENSITY AND STERIC ANOMALIES

For most oceanographic usage the full value of density or specific volume is unnecessary. Because the maximum variation in magnitude over the oceanic range of salinity, temperature, and pressure is only 7%, a considerable improvement in numerical resolution is achieved by using anomalies of specific volume and density. The specific volume (steric) anomaly δ(m^3/kg) and density anomaly γ(kg/m^3) are defined by

$$
\begin{aligned}
\delta(S, t, p) &= V(S, t, p) - V(35, 0, p) \\
\gamma(S, t, p) &= \rho(S, t, p) - 1000.0 \text{ kg/m}^3
\end{aligned}
\tag{17}
$$

where $V(35, 0, p)$ is given by a separate formula (Table 4). *Fofonoff and Millard* [1984] computed coefficients of the difference formulas needed to retain numerical precision in

TABLE 4. Coefficients for $V(35, 0, p)$ and $K(35, 0, p)$

Coefficient

$$
\begin{aligned}
V(35, 0, p) &= V(35, 0, 0)[1 - p/K(35, 0, p)] \\
K(35, 0, p) &= E_{35} + H_{35}p + M_{35}p^2 \\
\Delta K(S, t, p) &= K(S, t, p) - K(35, 0, p) \\
&= \Delta E_{35} + \Delta H_{35}p + \Delta M_{35}p^2
\end{aligned}
$$

$$
\begin{aligned}
E_{35} &= +21582.27 & \Delta E_{35} &= -1930.06 \\
H_{35} &= +3.359406 & \Delta H_{35} &= -0.1194975 \\
M_{35} &= +5.03217\text{E-}5 & \Delta M_{35} &= +3.47718\text{E-}5
\end{aligned}
$$

$$
\begin{aligned}
V(35, 0, 0) &= 972.662039 \times 10^{-6} \text{ m}^3/\text{kg} \\
\rho(35, 0, 0) &= 1028.106331 \text{ kg/m}^3 \\
\rho(35, 0, 0) - p(0, 0, 0) &= +28.263737 \text{ kg/m}^3
\end{aligned}
$$

Units: pressure, bars.

TABLE 5. Steric Anomaly and Specific Volume Differences Between Knudsen-Ekman and EOS 80 Equations of State for Seawater

	ΔV, $\times 10^{-9}$ m^3/kg		$\Delta\delta$, $\times 10^{-9}$ m^3/kg			
S/t		0°C	5°C	10°C	15°C	20°C
			p = 0 dbar			
34		−1.6	−2.3	−0.3	0.7	0.1
35	−18.9	0.0	−1.0	1.2	2.5	2.0
36		1.4	0.1	2.4	4.0	3.5
			p = 2000 dbar			
34		−1.6	−0.3	4.3	6.8	6.4
35	−43.4	0.0	1.2	5.9	8.6	8.2
36		1.4	2.4	7.3	10.1	9.8
			p = 4000 dbar			
34		−1.6	−0.9	3.4	5.4	5.1
35	−41.6	0.0	0.7	5.1	7.1	6.9
36		1.5	2.0	6.5	8.7	8.5

$\Delta V = V_{\mathrm{KE}} - V_{\mathrm{EOS\,80}}$

$\Delta\delta = \delta_{\mathrm{KE}} - \delta_{\mathrm{EOS\,80}}$

evaluating δ and γ. Given the two anomalies, the values of specific volume V and density ρ are recoverable to higher precision than from the full formula for a typical single-precision computation using a 32-bit computer word length. The increased numerical resolution is particularly important in computation of static stability where small vertical gradients of density must be resolved.

A significant departure from historical usage is taken in the definition of density anomaly γ. Previously, a specific gravity anomaly σ_t [*Knudsen*, 1901], defined as

$$\sigma_t = 1000\ (\rho(S, t, 0)/\rho(0, t_{\max}, 0) - 1) \tag{18}$$

was used, where $t_{\max}$ is the temperature of the density maximum of pure water at atmospheric pressure. The density anomaly γ, corresponding to σ_t, is

$$\gamma(S, t, 0) = \rho(0, t_{\max}, 0)(1 + \sigma_t/1000) - 1000.0\ \mathrm{kg/m^3} \tag{19}$$

Using the accepted value of $\rho(0, t_{\max}, 0) = 999.975$ kg/m^3 [*UNESCO*, 1974], (19) becomes

$$\gamma(S, t, 0) = 0.999975\ \sigma_t - 0.025\ \mathrm{kg/m^3} \tag{20}$$

The new definition of density anomaly yields values of γ that are numerically lower (the dimensions are different) than σ_t by a nearly constant amount. The difference must be taken into account for comparison of properties on potential density surfaces and similar graphical or numerical displays using σ_t as a variable. Steric anomalies computed from the Knudsen-Ekman equation of state [*Fofonoff*, 1962] differ by less than the precision of measurement from EOS 80 values. A comparison is given in Table 5. Over most of the open ocean range of variation, the differences are 2×10^{-9} m^3/kg or less and affect dynamic height or geopotential anomaly calculations by about 1 dynamic centimeter (0.1 J/kg) or less. As errors of measurement and random variations produced by internal waves are greater, the new salinity scale and equation of state do not significantly affect dynamic computations in agreement with the assessment of *Reid* [1958].

SPECIFIC HEAT C_p

Specific heat C_p is defined as the heat in Joules required to raise the temperature of 1 kg of seawater 1°C at constant pressure.

Millero et al. [1973] measured C_p for standard seawater, diluted with pure water or concentrated by evaporation, over a chlorinity range of 0–22‰ and a temperature range of 5–35°C. The measurements agreed with earlier determinations to within 2 J/(kg°C), except at low temperatures, where differences from values of *Cox and Smith* [1959] were as high as 6 J/(kg°C).

The formulas given by *Millero et al.* [1973] were converted to salinity by using the factor 1.80655 consistent with PSS 78 [*UNESCO*, 1981*c*, p. 118] for the 1-atmosphere terms.

No direct measurements of specific heat at elevated pressure are available for seawater. The pressure dependence is computed by integration of the thermodynamic relationship

$$\frac{\partial C_p}{\partial p} = -T\,\frac{\partial^2 V}{\partial t^2} \tag{21}$$

i.e.,

$$C_p(S, t, p) = C_p(S, t, 0) - \int_0^p (t + 273.15)\,\frac{\partial^2 V}{\partial t^2}\,dp \tag{22}$$

Using $V = V_0(1 - p/K)$, the derivative can be expressed in terms of V_0, K, and derivatives with respect to temperature. As pressure enters only in terms of the form p^m/K^n, where m and n are integers, the entire integral can be evaluated explicitly in terms of quadratic trinomial integrations of the form

$$I_n{}^m = \int_0^p p^m/K^n\,dp \tag{23}$$

TABLE 6. Precision of Data for Specific Heat of Seawater

	Minimum	Interval	Maximum	Number of Values	Total	Standard Deviation
			C(S, t, 0) from Millero et al. [1973]			
Chlorinity	0	variable	22‰	13	52	0.5 J/(kg°C) at 5°C
t	5	10	35°C	4		0.2 J/(kg°C) at 15-35°C
			ΔC_1 (from EOS 80)			
t	−2	2	42°C	23		
p	50	50	1000 bars	20	460	0.074 J/(kg°C)
			ΔC_2 (from EOS 80)			
S	5	5	40	8		
t	0	5	40°C	9		
p	100	100	1000 bars	10	720	0.062 J/(kg°C)

$C_p(S, t, p) = C_p(S, t, 0) + \Delta C_1(0, t, p) + \Delta C_2(S, t, p)$

TABLE 7. Specific heat of Seawater $C_p(S, t, p)$ in J/(kg°C)

	A	B	C
t^0	4127.4	−7.643575	+0.1770383
t^1	−3.720283	+0.1072763	−4.07718E-3
t^2	+0.1412855	−1.38385E-3	−5.148 E-5
t^3	−2.654387E-3		
t^4	+2.093236E-5		
	D	E	F
t^0	−4.9592 E-1	+4.9247 E-3	−1.2331E-4
t^1	+1.45747E-2	−1.28315E-4	−1.517 E-6
t^2	−3.13885E-4	+9.802 E-7	+3.122 E-8
t^3	+2.0357 E-6	+2.5941 E-8	
t^4	+1.7168 E-8	−2.9179 E-10	
	G	H	I
t^0	+2.4931 E-4	−2.9558 E-6	+9.971E-8
t^1	−1.08645E-5	+1.17054E-7	
t^2	+2.87533E-7	−2.3905 E-9	
t^3	−4.0027 E-9	+1.8448 E-11	
t^4	+2.2956 E-11		
	J	K	M
t^0	−5.422 E-8	+5.540 E-10	−1.4300E-12
t^1	+2.6380E-9	−1.7682E-11	
t^2	−6.5637E-11	+3.513 E-13	
t^3	+6.136 E-13		

$$\begin{aligned} C_p(S, t, p) = {} & A + BS + CS^{3/2} \\ & + (D + ES + FS^{3/2})p \\ & + (G + HS + IS^{3/2})p^2 \\ & + (J + KS + MS^{3/2})p^3 \end{aligned}$$

Coefficients are polynomials in temperature t. Units: salinity, PSS 78; temperature, °C; pressure, bars.

[*Gradsteyn and Ryzhik*, 1965]. In computing the pressure dependence the expressions were evaluated explicitly in double precision to serve as an independent check on direct computation by finite differencing and numerical integration of (22).

Tables of specific heat values were generated from the exact formulas and polynomial expressions fitted to the tables in two steps. The temperature and pressure dependence at $S = 0$ was evaluated by fitting to tables of $\Delta C_1 = C_p(0, t, p) - C_p(0, t, 0)$.

The salinity dependence was obtained by fitting additional terms to a table of

$$\Delta C_2 = [C_p(S, t, p) - C_p(S, t, 0)] - [C_p(0, t, p) - C_p(0, t, 0)]$$

so that

$$C_p(S, t, p) = C_p(S, t, 0) + \Delta C_1(0, t, p) + \Delta C_2(S, t, p) \tag{24}$$

Standard deviations of residuals from the fit are given in Table 6. The polynomial fits for ΔC_1 and ΔC_2 together with the polynomials for $C_p(S, t, 0)$ given by *Millero et al.* [1973] provide specific heat values over the full range of salinity temperature and pressure. The coefficients are given in Table 7. The polynomial terms giving the pressure dependence were fitted with higher precision so that the 1-atmosphere portion can be replaced without having to refit the pressure terms computed from EOS 80. Tables of C_p are given by *Fofonoff and Millard* [1984].

ADIABATIC LAPSE RATE Γ

The adiabatic lapse rate $\Gamma(S, t, p)$, defined as the change of temperature per unit pressure change for an adiabatic displacement, is required for computation of static stability and comparison of water types in the ocean. From thermodynamic considerations the lapse rate is given by

$$\Gamma(S, t, p) = (T/C_p)\frac{\partial V}{\partial t} \tag{25}$$

and can be computed from the equation of state and specific heat C_p.

Bryden [1973] developed a formula for Γ from *Bradshaw and Schleicher*'s [1970] measurements of thermal expansion of seawater. As these data were a subset of their thermal expansion data for EOS 80, the agreement with the new equation can be expected to be good. *Caldwell and Eide* [1980] estimated adiabatic lapse rates in seawater by measuring directly the temperature change produced by rapid changes of pressure. The two methods agree within the estimated uncertainties. However, the accuracies of the two techniques are not sufficient to permit a critical comparison.

For comparison with the *Bryden* [1973] and *Caldwell and Eide* [1980] formulas, tables of Γ were computed from the thermodynamical equations by using EOS 80 and fitted by standard least squares regression techniques with polynomials in S, t, and p. Sufficient terms were included to reduce the

TABLE 8. Adiabatic Lapse Rate $\Gamma_{\mathrm{EOS\,80}}$

	A	B	C	D
t^0	−4.4017	+0.264117	−6.298 E-3	−2.926 E-5
t^1	+1.159313	−8.4901E-3	+1.3742E-4	
t^2	−1.42752E-2	+9.5714E-5	+2.4153E-6	
t^3	+1.94206E-4	−3 483 E-7	−1 5540E-8	
t^4	−4.635 E-7	−7.244 E-9		
t^5	−1.2660 E-8			
	E	F	G	H
t^0	+2.4624 E-2	−2.027 E-4	+4.4614E-6	+2.9171E-7
t^1	−9.5979 E-4	+9.5596E-6	−3.7257E-7	−4.5887E-9
t^2	+1.8232 E-5	−2.283 E-7	+8.3866E-9	
t^3	−3.0623 E-7	+4.133 E-9	−9.8194E-11	
t^4	+3.5496 E-9	−5.392 E-11		
t^5	−2.3005 E-11	+3.717 E-13		
	I	J	K	
t^0	−4.9799 E-6	+3.965 E-8	−1.9155E-9	
t^1	+2.0840 E-7	−2.251 E-9	+6.5229E-11	
t^2	−3.5905 E-9	+4.143 E-11		
t^3	+6.4237 E-11	−4.234 E-13		
t^4	−5.9233 E-13			
	L	M		
t^0	−1.1437 E-9	+8.0102E-12		
t^1	+4.8665 E-11			
t^2	−1.1803 E-12			
t^3	+1.3351 E-14			
	N			
t^0	+3.5773 E-13			
t^1	−9.0147 E-15			

$$\begin{aligned} \Gamma(S, t, p) = {} & A + BS + CS^{3/2} + DS^2 \\ & + (E + FS + GS^{3/2} + HS^2)p \\ & + (I + JS + KS^{3/2})p^2 \\ & + (L + MS)p^3 \\ & + (N)p^4 \end{aligned}$$

Units: salinity, PSS 78; temperature, °C; pressure, bars; Γ, 10^{-4}°C/bar.

TABLE 9. Comparison of Lapse Rate Γ at $S = 35$, $t = 0$°C

Pressure, bars	$\Gamma_{BR} - \Gamma_{EOS\,80}$, °C/bar	$\Gamma_{CE} - \Gamma_{EOS\,80}$, °C/bar
0	7.8×10^{-6}	28.5×10^{-6}
200	4.2	23.5
400	−0.5	15.7
600	−5.0	8.6
800	−8.7	4.8
1000	−11.7	6.2

BR — *Bryden* [1973]; CE — *Caldwell and Eide* [1980].

fitting errors to about 0.1×10^{-7}°C/dbar. The coefficients are given in Table 8. Bryden estimated his fitting error to be about 3.4×10^{-7}°C/dbar and the uncertainty of the estimate of Γ to range from 27 to 53×10^{-7}°C/dbar or about 2% of Γ. Caldwell and Eide obtained a fitting error of 5.5×10^{-7}°C/dbar corresponding roughly to their estimated uncertainty. A comparison with Γ_{EOS80} at $S = 35$ is made in Table 9. The differences are within the rather conservative limits given by Bryden but large compared with the fitting errors. Bryden's values are closer to Γ_{EOS80}, as expected, but show a similar trend with pressure as the Caldwell and Eide residuals.

Because the differences in lapse rates yield adiabatic temperature changes that are within the precision of temperature measurements ($2\text{–}5 \times 10^{-3}$°C), *Fofonoff and Millard* [1984] selected the *Bryden* [1973] formula for computing lapse rates and potential temperatures. The introduction of a new formula would yield insignificant improvement in accuracy of potential temperatures and may be in need of further revision in the near future if the differences between the directly measured lapse rates of *Caldwell and Eide* [1980] and the computed rates are confirmed.

POTENTIAL TEMPERATURE θ

Potential temperature $\theta(S, t, p, p_r)$ is defined to be the temperature an element of seawater would have if moved adiabatically and with no change of salinity from an initial pressure p_0 to a reference pressure p_r that may be greater or less than p_0. This definition is more general than the classical definition given by *Helland-Hansen* [1912] and requires an explicit choice of the reference pressure p_r. However, it simplifies the computation of static stability, Brunt-Väisälä frequency, and potential density or steric anomaly for comparison of water types.

Under this definition the potential temperature is given by

$$\theta(S_0, t_0, p_0, p_r) = t_0 + \int_{p_0}^{p_r} \Gamma(S, \theta(S_0, t_0, p_0, p), p)\, dp \tag{26}$$

where Γ is the adiabatic lapse rate.

Fofonoff [1977] adopted a fourth-order Runge-Kutta integration scheme to evaluate the implicit integral in (26). He showed that the potential temperature is given to a numerical precision of 0.1×10^{-3}°C for the maximum integration step of 10,000 decibars.

Thus the integration can be carried out efficiently in a single step, making it unnecessary to develop separate formulas for potential temperature. A comparison of potential temperatures computed with the Runge-Kutta algorithm by using *Bryden*'s [1973] and *Caldwell and Eide*'s [1980] formulas is made in Table 9.

Table 10 contains differences from potential temperatures computed from EOS 80 by using the thermodynamical relationships. Agreement at 10°C is better for the direct measurements of *Caldwell and Eide* [1980]. At 0°C, agreement is better with values obtained from *Bryden*'s [1973] formula. As the differences between the Bryden formula and EOS 80 in the oceanic range are comparable to the precision of temperature measurements made at sea, no real improvement in accuracy is gained by introducing a more precise formula for Γ. As a consequence, *Fofonoff and Millard* [1984] recommended using the simpler Bryden formula for Γ for computation of potential temperature.

The change of salinity scale to PSS 78 produces negligible changes in Γ or potential temperature θ. The maximum difference in θ was estimated to be 0.3×10^{-3}°C, using *Bryden*'s [1973] Γ for $p_0 = 10{,}000$ dbar, $p_r = 0$ dbar.

BRUNT-VÄISÄLÄ FREQUENCY

An element of seawater displaced adiabatically and isentropically to z from its equilibrium level z_0 in a stratified layer will have a density difference $\Delta\rho$ from its surroundings of

$$\Delta\rho = \rho(S_0, \theta(S_0, t_0, p_0, p), p) - \rho(S, t, p) \tag{27}$$

and will be acted upon by a restoring buoyancy force F per unit mass, given by

$$F = -(\Delta\rho/\rho)g \tag{28}$$

where g is gravity. The mass element will be subjected to a vertical acceleration $\ddot{z}$, where

$$\begin{aligned} \ddot{z} &= +F = -(\Delta\rho/\rho)g \\ &= -N^2(z - z_0) = -N^2\Delta z \end{aligned} \tag{29}$$

and, using the hydrostatic relationship $\Delta p = -\rho g \Delta z$,

$$N^2 = +(g/\rho)\Delta\rho/\Delta z = -g^2\Delta\rho/\Delta p \tag{30}$$

The Brunt-Väisälä frequency N is a measure of the stratification and the high-frequency limit for internal waves that can be supported by the vertical potential density gradient.

The dependence of N^2 on thermal expansion and saline contraction coefficients is obtained by expanding (27) in a

TABLE 10. Potential Temperature Differences $\theta - \theta_{EOS\,80}$ for $S = 35$ and Reference Pressure $p_r = 0$ bar

Pressure, bars	0°C		10°C	
	CE, × 10^{-3}°C	BR, × 10^{-3}°C	CE, × 10^{-3}°C	BR, × 10^{-3}°C
200	−5.3	−1.3	−1.7	1.0
400	−9.2	−1.8	−2.0	2.0
600	−11.8	−1.6	−1.1	3.1
800	−13.4	−0.7	+0.1	4.3
1000	−15.1	+0.5	+0.8	5.5

BR — *Bryden* [1973]; CE — *Caldwell and Eide* [1980].

Taylor series about p_0 as follows

$$\rho(S_0, \theta, p) = \rho(S_0, t_0, p_0) + \frac{\partial \rho}{\partial t}\Delta\theta + \frac{\partial \rho}{\partial p}\Delta p + O(\Delta p^2)$$

$$\rho(S, t, p) = \rho(S_0, t_0, p_0) + \frac{\partial \rho}{\partial S}\Delta S + \frac{\partial \rho}{\partial t}\Delta t + \frac{\partial \rho}{\partial p}\Delta p + O(\Delta p^2) \tag{31}$$

so that

$$\frac{\Delta\rho}{\Delta p} = -\left[\frac{\partial \rho}{\partial S}\frac{\Delta S}{\Delta p} + \frac{\partial \rho}{\partial t}\left(\frac{\Delta t}{\Delta p} - \Gamma\right)\right] + O(\Delta p) \tag{32}$$

The compressibility terms cancel exactly.

For computational convenience the expression for N^2 is converted to use specific volume anomaly, i.e.,

$$\begin{aligned} N^2 &= -g^2\Delta\rho/\Delta p = +\rho^2 g^2 \Delta V/\Delta p = \rho^2 g^2 \Delta\delta/\Delta p \\ &= \rho^2 g^2[\delta(S_0, \theta(S_0, t_0, p_0, p), p) - \delta(S, t, p)]/\Delta p \end{aligned} \tag{33}$$

The use of potential steric anomaly δ to compute N provides the maximum numerical resolution and reduces the computation to two evaluations of steric anomaly. *Fofonoff and Millard* [1984] give algorithms for computing ρ, V, δ. The procedure can be elaborated by referencing a series of observations over a suitable pressure interval to a single pressure and fitting a least squares curve to the potential steric anomalies. Such a method [*Bray and Fofonoff*, 1981] has the advantage of vertical averaging to improve the estimate of N^2.

DISCUSSION

The need to establish standards for measurement of salinity with a precision matching the sensitivity of present-day conductivity bridges has forced abandonment of scales based on natural seawaters in favor of a more rigidly and precisely defined practical scale referenced to a conductivity standard. The practical scale will continue to be used until measurement techniques advance in precision and resolution beyond the present standards. As salinity is combined with temperature and pressure to compute density in many applications, the relationship between conductivity and density will need to be explored further to establish the range of compositional variations in the world oceans and to establish base lines for monitoring time-dependent changes in the relationship.

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N. P. Fofonoff, Department of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, MA 02543.

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